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Human Capital and its incidence in the Solow growth model

1. Introduction

Regory Mankiw, David Romer and David Weil (MRW, 1992) conclude in their work that by adding the component of human capital to the Solow growth model, international differences in income per capita are better understood. The authors named it the augmented Solow model. I begin this paper by explaining why MRW use logs in their study. After a brief introduction of the Solow Model I analyse the process leading to their conclusion using statistical software known as Eviews and showing that the authors' deduction is solid. Finally, this paper states its own conclusion.

After the concluding statement, I created an appendix containing the different tables generated by Eviews.

2. Use of Logs in the regression

The authors use what is known as panel data in their analysis (Cartel Hill, Lim and Griffiths, 2008). They study how GDP per capita is affected in a set of different countries, namely OCDE, intermediaries and Non-Oil countries, given the variations over time in a series of explanatory variables. Heteroskedasticity is normally found when confronting this type of data. According to Cartel Hill, Lim and Griffiths (2008, p.198), heteroskedasticity ensues when "[…] the variances for all observations are not the same […]". Using the White test in Eviews, I verified the evidence of heteroskedasticity with the following null and alternative hypothesis:

$H_{o=}$ homoskedasticity exists in the data

H_1: Heteroskedasticity exists in the data

The F-Statistic and the Obs* R-squared tests are very significant in Non-Oil and Intermediaries, rejecting the null hypothesis at a level of significance of 0.01 and 0.10 respectively, whereas for OCDE countries the P-value is clearly higher. Nonetheless, for simplicity and given the results obtained using Eviews for the rest of nations, I decided to reject the null hypothesis accepting the existence of heteroskedasticity for the whole set of data, which violates the Gauss-Markov theorem because the following assumption no longer holds:

$Var(\varepsilon_i) = Var(Y_i) = \sigma^2$

This suggests a problem with the Ordinary Least Squares (OLS) method used by MRW, because it will no longer be BLUE (Best Linear Unbiased Estimator). The repercussion of using OLS under heteroskedasticity is obtaining faulty standards errors, invalidating T-statistic and F statistic tests. (Cartel Hill, Lim and Griffiths, 2008) Such tests are used to obtain the P-values on the MRW's paper.

The authors use a Log-Log transformation, which involves the conversion into logs of the *regressors* and the *regressands*. (Cartel Hill, Lim and Griffiths, 2008). By using this the model is linearized and often the variances become homoskedastic. (Picconi and Reynolds, 2013). Strictly speaking, the interpretation given by the coefficients acquired using the log-log model states that an expected variation in Y occurs when there is an alteration in the controlled variables, in other words, coefficients are considered elasticities.

3. Solow model of growth

3.1. Text book Solow Model

The Solow model uses The Cobb-Douglas production function with constant returns to scale of output to inputs, but diminishing returns to individual factors of production.

(1) $Y(t) = K(t)^{\alpha} (A(t) L(t))^{1-\alpha} \qquad 0 < \alpha < 1$

The model assumes labor and capital are endogenous factors, but population growth and investment, which equals savings, are exogenous. The constant α is capital's share in income, which is estimated roughly at 1/3. Inserting 1/3 into *(α/1- α)*, the elasticities of savings and population growth factors can be found, which are 0.5 and -0.5 respectively

Taking logs, the econometric model is as follows:

(2) $Ln(\text{GDP per capita}) = \beta 0 + \beta 1 Ln(sY) + \beta 2 Ln(n + g + d)$

Where *sY* equals investment and *n+ g +d* is the depreciation of capital.

By using OLS, MRW obtain the coefficients, which explain the degree of impact that investment and depreciation rate have on GDP per capita. (TABLES 1, 2 and 3)

MRW find that investment and population growth affect income per worker positively and negatively respectively. The adjusted R-squared, the coefficient of determination which measures the degree of linear correlation between the dependent variable and the independents, is highly significant. It is able to explain that roughly 59% of the variations in GDP per worker in Non-oil and Intermediaries countries can be attributed to those variables; however OCED countries do not perform well. Another divergence is found when assessing the constant α given by the coefficients. It is not consistent with the implied 1/3 estimated by

MRW, as it is much higher. Moreover, when imposing the constant on the restricted model the adjusted R-squared decreases. Another negative outcome is the extraordinary level of the coefficients, which predict a much larger impact of savings and population growth on GDP per capita.

3.2. The Augmented Solow Model

The same variables apply to the augmented Solow Model, but the authors add a new variable to the equation, human capital. The new production function is as follows:

(3) $Y(t)= K(t)^{\alpha} H(t)^{\beta} (A(t) L(t))^{1-\alpha\ \beta}$ $\alpha + \beta < 1$

The notations are the same as before, where H is the stock of human capital. There are decreasing returns to all capital. Savings have to be invested in physical capital as well as human, so the functions are Sk and Sh respectively.

Taking logs, the new econometric model is as follows:

(4) $\text{Ln (GDP per capita)} = \beta 0 + \beta 1\text{Ln}(Sk) + \beta 2\text{Ln}(n + g+d) +\beta 3\text{Ln}(Sh)$

In this case, assuming the capital's share of income $\alpha = \beta= 1/3$, the elasticities are 1 for human capital as well as physical, and -2 for population growth.

By estimating the coefficients using OLS, MRW find that investment and school have positive signs, and population growth affects GDP per capita negatively. (TABLES 4, 5 and 6)

In this case the coefficients are much lower, eliminating the anomalies seen previously. It also improves the goodness of fit as the adjusted R-squared also displays a larger number, explaining that almost 80% of the changes in GDP per capita can be attributed to the hand right variables, therefore the performance of the regression line is better. However, the case of OCDE countries is less convincing, as the value of the coefficient of determination is barely 25%. When restricting the regression the implied values of α and β are around 1/3, again OCDE countries do not perform as good as the rest.

4. Wald test in the augmented Solow model

In order to test the significance of the newly added coefficient; I used the Wald test with the following hypothesis:

H_0: coefficient 4= 0

In our case, *school* is the coefficient number four in the tables generated using Eviews. The aforementioned Null hypothesis is performed, and the P-value of the T- statistic is zero for Non-Oil and Intermediaries countries, so in these cases the null hypothesis is rejected. In the case of OECD nations, given the P-value of 0.0175, the null hypothesis is rejected at a level of significance of 0.05. Hence the probability of obtaining such a value, in this case that the coefficient equals zero, is practically improbable. Consequently the coefficient *school* is proved useful in the augmented Solow model as it has an impact on GDP per capita.

5 Conclusion

I agree with the conclusion that by adding human capital to the regression, income per capita is better understood, as the adjusted R-squared increases dramatically to almost 80%, which means that nearly 80% of the variations in GDP per capita are due to fluctuations in the inputs, namely savings, population growth and human capital. By adding human capital the performance of physical capital improves, since their elasticity with respect to the dependant variables rises and they are in consonance with the implied values of α and β . Using output to invest in physical and human capital consequently increases the GDP per capita, enhancing consumption. Yet, OECD countries remain largely unexplained because the Adjusted R-squared in both the text book Solow model and the augmented Solow model can hardly explain the differences in income in such nations by using the inputs mentioned in the beginning of this conclusion. They also displayed anomalies in comparison with the remaining countries when performing the White test for heteroskedasticity and the Wald test. Therefore there is room for further research into this topic.

6 Appendix

Tables for the text book Solow model

Table 1: Non-Oil countries

Dependant Variable: Log (RGDP per worker1985)

Variable	Coefficient
C	5.429883
LOG(I_Y/100)	1.424014
LOG(0.05+POPGROWTH/100)	-1.989774
R-squared	0.600865
Adjusted R-squared	0.592462

Table 2: Intermediaries

Dependant Variable: Log (RGDP per worker 1985)

Variable	Coefficient
C	5.345865
LOG(I_Y/100)	1.317553
LOG(0.05+POPGROWTH/100)	-2.017199
R-squared	0.598908
Adjusted R-squared	0.587767

Table 3: OECD countries

Dependant Variable: Log (RGDP per worker 1985)

Variable	Coefficient
C	8.020607
LOG(I_Y/100)	0.499890
LOG(0.05+POPGROWTH/100)	-0.741921
R-squared	0.105926
Adjusted R-squared	0.011813

Tables for the augmented Solow Model

Table 4: Non-Oil countries

Dependant Variable: Log (RGDP per worker 1985)

Variable	Coefficient
C	6.844414
LOG(I_Y/100)	0.696709
LOG(0.05+POPGROWTH/100)	-1.745247
LOG(SCHOOL/100)	0.654459
R-squared	0.785636
Adjusted R-squared	0.778795

Table 5: Intermediaries

Dependant Variable: Log (RGDP per worker 1985)

Variable	Coefficient
C	7.791314
LOG(I_Y/100)	0.700367
LOG(0.05+POPGROWTH/100)	-1.499780
LOG(SCHOOL/100)	0.730549
R-squared	0.780692
Adjusted R-squared	0.771425

Table 6: OECD countries

Dependant Variable: Log (RGDP per worker 1985)

Variable	Coefficient
C	8.636893
LOG(I_Y/100)	0.276134
LOG(0.05+POPGROWTH/100)	-1.075506
LOG(SCHOOL/100)	0.767571
R-squared	0.352352
Adjusted R-squared	0.244411

7. References

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